Contribution of Charged Vector Bosons to the Photon Propagator*

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We consider the effect of charged vector bosons on the photon propagator, within the framework of the ξ-limiting formalism. The two cases, of vanishing and nonvanishing primary anomalous magnetic moment, are contrasted, and several experimental consequences are discussed.

THERE has been, of late, a considerable degree of interest in the electrodynamics of charged vector bosons. The ξ-limiting formalism devised by Lee and Yang[1] for extracting finite answers has had a measure of theoretical success, in that Lee[2] was able to calculate the quadrupole moment of the vector boson. The calculations of Lee were geared to the case of nonvanishing anomalous magnetic moment ($\kappa\neq0$). Very recently, however, Bernstein and Lee[3] have pointed out that the theory with $\kappa\neq0$ leads to a neutrino charge radius independent of the electric charge. Furthermore, the precise value of the charge radius cannot be determined until some procedure is found for defining the highly singular power series which occurs in the calculation. Bernstein and Lee are thus led to suggest that an acceptable theory can be constructed only for the case $\kappa=0$. However the neutrino form factor involves two unrenormalizable couplings; an inclusion of higher order weak effects[4] may bring in modifications which are as yet unknown. It seems desirable therefore to compare the predictions of the theories with $\kappa\neq0$ and $\kappa=0$ in situations which are purely electrodynamic in nature and where one deals with quantities more singular than the quadrupole moment.

In the present note we consider the effect of charged vector bosons on the photon propagator. These effects may be expressed in terms of a gauge invariant polarization tensor $\Pi_{\mu\nu}$ which we choose to define as follows:

$$D_{\mu\nu}'(k)=D_{\mu\nu}(k)+D_{\mu\rho}(k)\Pi^{\rho\sigma}(k)D_{\sigma\nu}(k)\,, \tag{1}$$

$$D_{\mu\nu}(k)=g_{\mu\nu}/k^2\,, \tag{2}$$

$$\Pi_{\mu\nu}(k)=(k^2g_{\mu\nu}-k_\mu k_\nu)\Pi(k^2)\,. \tag{3}$$

Here k is the 4-momentum of the photon and the $\Pi_{\mu\nu}$ defined above agrees with the usual definition[5] to order α (for a renormalizable theory!). The utility of our definition lies, of course, in the simple relationship between $\Pi(k^2)$ and the Källen-Lehmann[5] spectral function $\sigma(k^2)$.

$$\mathrm{Im}\Pi(k^2)=\pi k^2\sigma(k^2)\,. \tag{4}$$

The calculation of $\Pi(k^2)$ is messy but straightforward; as a purely technical point it may be mentioned that the use of dispersion relation techniques results in an enormous simplification. In fact, the complete gauge invariance of the absorptive part of $\Pi_{\mu\nu}$ enables us to dispense with all ξ-dependent terms in the Feynman rules given by Lee and Yang; the cutoff is simply introduced in the dispersion integrals at the threshold where the metric in Hilbert space ceases to be positive definite. Without further ado then, we quote the explicit answer

$$\Pi(k^2)=\frac{k^2}{\pi}\int_{4m^2}^{m^2(1+\xi^{-\frac{1}{2}})^2}\frac{\mathrm{Im}\Pi(x)dx}{x(x-k^2-i\epsilon)}\,, \tag{5}$$

where

$$\frac{\mathrm{Im}\Pi(x)}{x}=-\frac{\alpha}{6}\left(\frac{x-4m^2}{4x}\right)^{1/2}\left[\left(\frac{\kappa^2m^2}{4}\right)\frac{x}{m^6}+(1+3\kappa)\frac{1}{m^2}-(1+12\kappa+4\kappa^2)(1/x)-(12m^2/x^2)\right]. \tag{6}$$

Here m is the mass of the boson and since we deal with renormalized photon field operators, we have set $\Pi(0)=0$.

It is sufficient to confine our attention to the quantity $\Pi'(0)$. Clearly the most singular part of $\Pi(k)^2/k^2$ is identical to that of $\Pi'(0)(k^2\ll m^2\xi^{-1})$. As we shall see, this implies that to leading order $\Pi(k^2)/k^2=\Pi'(0)$.

Case I: $\kappa\neq0$.

We have

$$\Pi'(0)=-\frac{1}{m^2}\frac{\alpha}{48\pi}\left(\frac{\kappa^2}{\xi}\right)+\text{less singular terms.} \tag{7}$$

The inclusion of higher order proper graphs gives a *multiplicative* factor of the form

$$\sum_{n=0}^{\infty}a_n(\alpha\kappa^2/\xi^2)^n,\quad a_0=1 \tag{8}$$

if we retain only the most singular parts.[2]

Hence,

$$\Pi'(0)=-\frac{1}{m^2}\frac{\kappa\alpha^{\frac{1}{2}}}{48\pi}\cdot C+\frac{1}{m^2}O(\alpha\ln\alpha)\,, \tag{9}$$

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[1] T. D. Lee and C. N. Yang, Phys. Rev. **128**, 885 (1962).
[2] T. D. Lee, Phys. Rev. **128**, 899 (1962).
[3] J. Bernstein and T. D. Lee, Phys. Rev. Letters **11**, 512 (1963).
[4] G. Feinberg and A. Pais, Phys. Rev. **131**, 2724 (1963); **133**, B477 (1964); Y. Pwu and T. T. Wu, *ibid.* **133**, B778 (1964); M. A. B. Bég, Ann. Phys. (to be published).
[5] See, e.g., S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Row, Peterson, and Company, New York, 1961).

where

$$C=\lim_{x\to\infty} x^{1/2} \sum_{n=0}^{\infty} a_n x^n \qquad (10)$$

is an undetermined constant.

Case II: $\kappa=0$.

Here we obtain

$$\Pi'(0)=-\frac{1}{m^2}\left(\frac{\alpha}{12\pi}\right)\ln\xi^{-1}+\text{finite terms.} \qquad (11)$$

If we accept a conjecture in Ref. 3, the most singular parts of higher order proper graphs now give an *additive* factor

$$\alpha \sum_{n=1}^{\infty} b_n(\alpha/\xi)^n \qquad (12)$$

and we obtain

$$\Pi'(0)=\frac{1}{m^2}\frac{\alpha\ln\alpha}{12\pi}+\frac{1}{m^2}O(\alpha). \qquad (13)$$

Equation (9) displays the difficulty encountered in a purely electrodynamic situation when $\kappa\neq 0$. One cannot say that the result is physically inadmissible, however, until one knows how to sum the series in Eq. (8), one cannot predict any numbers either. It may be possible in the future to calculate the constant C by some peratization like procedure.[4] In the meantime our inability to make numerical predictions for a large number of physically interesting quantities appears as the sole basis for rejecting the theory with $\kappa\neq 0$. Needless to say an experimental measurement of the magnetic moment of W particles is highly desirable.

We turn now to the question of experimental measurability of the effect we have calculated. Since the mass of the boson is expected to be fairly large ($\sim$1 BeV), high-energy electron-electron and electron-positron scattering appear to be most suitable for the purpose at hand.

Let us first consider Møller scattering. The relevant matrix element may be exhibited as

$$M\sim\alpha\bar{u}(p_1')\gamma_\mu u(p_1)\bar{u}(p_2')\gamma^\mu u(p_2)\left[\frac{1}{t}+\frac{1}{t}\Pi(t)\right]$$
$$+\alpha\bar{u}(p_2')\gamma_\mu u(p_1)\bar{u}(p_1')\gamma^\mu u(p_2)\left[\frac{1}{u}+\frac{1}{u}\Pi(u)\right]. \qquad (14)$$

Here p_i and p_i' ($i=1, 2$) are the initial and final electron momenta; $t=(p_1'-p_1)^2$, $u=(p_2'-p_1)^2$ are the direct and exchange momentum transfers, respectively. For the sake of consistency Eq. (14) should be corrected for the charge structure of the electrons, the coupling to the anomalous moment and the exchange of an additional photon. These corrections are well known and need not be discussed here.[6]

For very large $|z|$ ($z=u$ or t), the contributions to $(1/z)\Pi(z)$ arising from pair fields whose electrodynamics is renormalizable are expected to be damped.[7] However, within the domain of validity of Eq. (9), ($|z|<m^2\kappa^{-1}\alpha^{-1/2}$) or of Eq. (13), ($|z|<m^2\alpha^{-1}$) the vector boson contribution to $z^{-1}\Pi(z)$ behaves as a constant. The vector boson contribution may therefore be measurable in an experiment in which the momentum transfers are somewhat larger than m.

The above discussion can be extended to Bhabha scattering by making some trivial substitutions. The *fractional* change in the differential cross section for both Møller and Bhabha scattering is given by

$$\left(\frac{d\sigma}{d\Omega}\right)^{-1}\Delta\left(\frac{d\sigma}{d\Omega}\right)=+\frac{\kappa C(\alpha)^{1/2}}{4\pi}\left(\frac{E}{m}\right)^2\frac{\sin^2\theta}{3+\cos^2\theta},\quad \kappa\neq 0, \qquad (15a)$$

$$=-\frac{\alpha\ln\alpha}{\pi}\left(\frac{E}{m}\right)^2\frac{\sin^2\theta}{3+\cos^2\theta},\quad \kappa=0, \qquad (15b)$$

where E is the electron energy and θ the scattering angle, all quantities being in the center-of-mass frame. Note that the extreme relativistic limit has been taken. For the sake of numerical orientation let us take $m=1$ BeV and consider scattering at 90°. The vector boson contribution to the differential cross section overtakes the resonant pion pair contribution at $E\sim 1.28$ BeV, the muon pair contribution at $E\sim 1.25$ BeV, and the electron pair contribution at $E\sim 2.5$ BeV.

We have also considered the effect of vector bosons on the level shift in muonic atoms and on the magnetic moment of the muon. These turn out to be extremely small; the following discussion is nevertheless included for the sake of completeness.

The level shifts are confined only to S states, in leading order. The important $3D_{5/2}-2P_{3/2}$ line in muonic phosphorous is therefore unaffected.[8] The fractional shift of a $2P-1S$ line is given by

$$\Delta E/E=(\kappa C)(Zm_\mu/m)^2 0.32\times 10^{-6},\quad \kappa\neq 0, \qquad (16a)$$
$$=(Zm_\mu/m)^2 0.54\times 10^{-6},\quad \kappa=0. \qquad (16b)$$

($Z\equiv$ charge number of nucleus.)

The contribution to the magnetic moment of the muon, calculated most conveniently by using some results due to Durand,[9] is given by

$$\Delta\kappa_\mu=(\kappa C)(0.081)(m_\mu/m)^2(\alpha/\pi)^2,\quad \kappa\neq 0, \qquad (17a)$$
$$=(0.14)(m_\mu/m)^2(\alpha/\pi)^2,\quad \kappa=0 \qquad (17b)$$

in units of μ magnetons. The expression for $\kappa=0$ is of

[6] See Y. S. Tsai, Phys. Rev. **120**, 269 (1960). A more complete calculation, however, is very desirable.

[7] In perturbation theory the damping goes as $|z|^{-1}\log|z|$.

[8] A. Petermann and Y. Yamaguchi, Phys. Rev. Letters **2**, 359 (1959).

[9] L. Durand, III, Phys. Rev. **128**, 441 (1962).

the same order of magnitude as the weak interaction contribution to κ_μ calculated by Pietschmann and Segrè.[10] Clearly it will take some time before these contributions can be verified experimentally.

[10] H. Pietschmann (unpublished); G. Segrè, Phys. Letters **7**, 357 (1963).

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K^-p Charge-Exchange Scattering at 1.80 GeV/c

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Measurements have been made of the differential and total K^-p charge-exchange cross section at 1.80 GeV/c. The total cross section found was 1.55±0.09 mb. Terms including $\cos^{10}\theta_{K^0K^-}$ were required in fitting the $\bar{K}^0$ angular distribution.

A STUDY has been made of the reaction

$$K^-+p\rightarrow\bar{K}^0+n \qquad (1)$$

using film obtained from an exposure of the Lawrence Radiation Laboratory 72-in. hydrogen bubble chamber to a separated K^- beam. This beam had a central momentum of 1.80 GeV/c (2150 MeV total c.m. energy and 785 MeV/c c.m. K^- momentum) and a 6% momentum spread. The distribution of the $\bar{K}^0$ production angle in the center-of-mass system is strongly peaked forward. This contrasts with the pronounced backward peak observed[1,2] at lower momenta (760–1220 MeV/c). The result reported here continues the tendency first noticed at 1.53 GeV/c incident momentum,[2] for the backward peak to be replaced by a forward one.

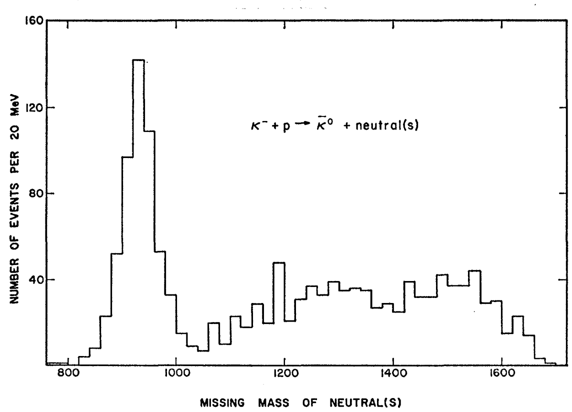

FIG. 1. Mass distribution of the neutrals in the reaction $K^-+p\rightarrow\bar{K}^0$+neutral(s) for incident K^- momentum 1.80 GeV/c (2150 MeV/c total c.m. energy). $K^-+p\rightarrow\bar{K}^0+n$ events are contained in the peak at the neutron mass.

The events appeared in the chamber as zero prong interactions accompanied by a decay V. A large fraction of the Λ hyperons produced in zero prong interactions was rejected on inspection of the ionization of the positive V track. For the measured events, the kinematical fits were usually adequate to identify the decaying particle as a Λ or a K_1^0. The particle was called a K_1^0 decaying *via* $K_1^0\rightarrow\pi^++\pi^-$ if the hypothesis fitted with a $\chi^2\leqslant 30.0$ (3 constraints); 2.5% of the sample remained ambiguous between Λ and K_1^0 on the basis of both kinematical fit and ionization.

Figure 1 shows the distribution of mass of the neutral particle(s) produced in addition to the $\bar{K}^0$. For 16 events (3.5% of the sample), the ambiguity between the $\bar{K}^0n$ final state and events in which additional pions were produced could not be resolved on the basis of the missing mass and its error.

To reduce scanning biases, only events with K^0 track length in space greater than 0.5 cm were allowed in the sample. A correction factor was later applied to each angular interval to compensate for the loss. The number of events in the sample was also corrected as follows: (a) +2.5% due to over-all scanning loss (the scanning efficiency was found to be independent of scattering angle); (b) +3.0% due to events which were unmeasurable; (c) +1.0% due to K_1^0's incorrectly identified as Λ's in the preliminary ionization study; (d) +3.0% due to ambiguous events discussed above.

The beam flux was determined by a count of the τ-like decays of K^- in the same sample of film. A branching

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[1] W. Graziano and S. G. Wojcicki, Phys. Rev. **124**, 1868 (1962).
[2] M. Ferro-Luzzi, F. T. Solmitz, and M. L. Stevenson, *Proceedings of the 1962 International Conference on High Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 376.